Information Theory:
An Analysis and Design Tool for HCI

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ABSTRACT

Shannon's information theory, since its first introduction in 1948, has received much attention and successful applications in many domains. Apart from Fitts' law and Hick's law, which came out when experimental psychologists were enthusiastic about applying information theory to various areas of psychology, the relation of information theory to human-computer interaction (HCI) has not been clear. Even the two above-mentioned "laws" remain controversial in both psychology and HCI. On the other hand, in recent years, information theory has started to directly inspire or contribute to HCI research.

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This paper examines these information-theoretic applications and discusses the opportunities where information theory can be used to understand, design and optimize human-computer communication.

CCS CONCEPTS

• **Human-centered computing → HCI theory, concepts and models**.

KEYWORDS

HCI, information theory, model, performance, optimization, entropy, mutual information

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Entropy:

$$H(X) = -\sum_x p(x) \log_2 p(x).$$

where X is drawn according to the probability distribution $p(x) = P(X = x)$ and entropy $H(X)$ is measured in bit.

Mutual Information:

$$I(X;Y) = \sum_y \sum_x p(x,y) \log \frac{p(x,y)}{p(x)p(y)} = H(X) - H(X|Y)$$

where $p(x,y)$ is the joint probability function of X and Y, and $p(x)$ and $p(y)$ are the marginal probability distribution functions of X and Y, respectively.

Channel Capacity:

$$C = \max_{p(x)} I(X;Y)$$

where the maximum is taken over all possible input distributions $p(x)$.

Shannon's Theorem 17:

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

where B is the bandwidth and S/N is the signal-to-noise ratio.

INFORMATION THEORY CONCEPTS

The communication scheme proposed by Shannon [19] (Fig. 1) has pioneered the modern analysis of digital communication and established *entropy* as a relevant measure of information. It states that a *source* produces messages, modeled as a random variable X, which are adapted by an *encoder* and then sent over a *channel* and decoded by a *decoder* to the final *destination*. The input of the channel is X and the output of the channel to the *receiver* is Y. Since there might be *noise* in the channel, output Y does not always equal input X. The engineering process to transmit a source message X through the channel does not concern the semantic aspect of communication, but is only related to the probability of each possible outcome. Therefore, the channel is completely described by the probability $p(Y|X)$ of Y given X. The key notions that have largely been used in psychology and HCI are *entropy*, *mutual information* and *channel capacity* (see sidebar on page 2).

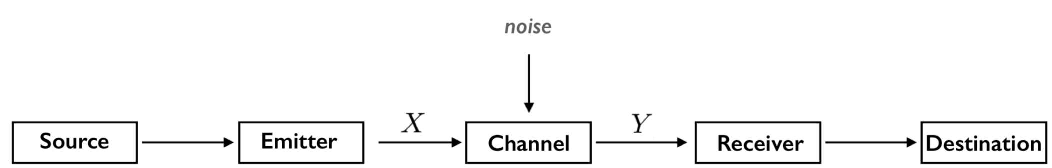

Figure 1. Shannon's communication scheme.

INFORMATION-THEORETIC APPLICATIONS IN HCI

What is Throughput in Fitts' Law?

One direct application of Shannon's information theory that has been proven useful and has been largely studied in HCI community is *pointing*. Among many experimental psychologists back in the 1950s [1], Fitts conceptualized the human motor system as a communication channel [6] and proposed an operationalized formula [2] to capture the relationship between movement time MT and what we call *index of difficulty* ID. Fitts [6] also derived the *Index of Performance* IP, which is computed by dividing ID by the empirically determined movement time MT: $IP = \frac{ID}{MT}$, to represent the participant's maximum rate. This notion was later borrowed by Mackenzie [14] as *throughput*, which in engineering is widely used to measure an effective speed of data transmission [3]. Building on Card et al. [3], Zhai [23] argued that Mackenzie's throughput "is an ill-defined concept" and instead described the constant b in Fitts' formula as the informational aspect of input performance. Since Fitts' law is an analogy to Shannon's Theorem 17 (see Gori et al. [7] and sidebar on page 2), information theory should provide a straightforward way to analyze throughput. The first author's thesis [11] introduces an information-theoretic notion of throughput that generalizes beyond Fitts' law by considering input as communicating what is in the user's head to the computer over a noisy channel. We demonstrate its consistency compared with two existing notions of throughput and extend it from a single input random variable to sequential input such as text entry, accounting for conditional probability and a stochastic input process.

[1] Most of these applications were summarized in a book by Attneave [1].

[2] Fitts' formula:

$$MT = a + b \times ID.$$

where a and b are empirically determined constants.

[3] Throughput captures the successfully transmitted information rate. It is computed by dividing the amount of successfully transmitted information $I(X;Y)$ by the average time T required to transmit such information and is measured in bits per second: $TP = I(X;Y)/T$.

How Relevant is Hick's Law for HCI?

Together with Fitts' law, Hick's law [8, 9] was introduced to HCI in the early 1980s as a contribution of psychology to the design of human-computer interfaces [4]. It has been used in a number of HCI contexts but its applications have been controversial: Some researchers found that the law did not apply to HCI tasks while others regard it as a fundamental law of interface design. Hick and Hyman originally studied the relationship between the number of alternate stimuli and choice-reaction time by considering each alternate stimulus as the message; the sensory-perceptual system as the channel and the participant as the receiver. Choice-reaction time increases with uncertainty, captured by the information-theoretic notion of entropy (sidebar on page 2). HCI researchers, however, have applied it to various scenarios, including expert users' decision time by Cockburn et al. [5] and novice users' visual search time by Soukoreff & Mackenzie [20]. Even though Sears et al. [18] showed the incompatibility of the law with visual search, Wobbrock & Myers [22] used it to model visual search time. In the design community (e.g. [10]), on the other hand, the law seems to work universally. Our review of the state of the art of Hick's law as well as the historical context of the choice-reaction paradigm leads to the conclusion that Hick's law is in fact not very relevant for HCI [11].

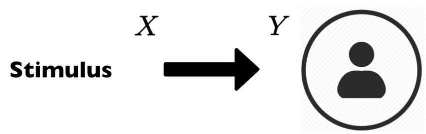

Figure 2. A model of communication system where users are the receivers [8, 9].

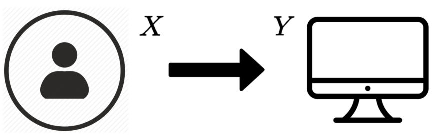

Figure 3. A model of communication system where users are the information source [2, 13, 15, 17].

A Human-Computer Communication Framework?

Several recent works have investigated the information transmission process from the user to the computer using the tools of information theory. Table 1 and Fig. 3 summarize these direct applications of information theory to HCI. We believe that these and the above-mentioned studies can be synthesized and expanded into a general human-computer communication framework. If we consider that the purpose of any input technique is to let users send information to the computer, the information-theoretic approach is well-suited to their characterization. For example, we have introduced the Bayesian Information Gain (BIG) framework [12] that quantifies the information in the user input to reduce the computer's uncertainty about the user's goal, expressed in a probabilistic model. One can use this framework to measure the information sent by the user to the computer. Moreover, by maximizing the expected information gain from the user's subsequent input through a proper choice of feedback, the computer can play a more active role in the interaction. By leveraging the notion of information gain, we can shift the balance of who is in control and investigate the notion of combining human intelligence with machine power.

Table 1. Recent HCI studies that leveraged the information-theoretic concepts.

Reference	*Area*	*Description*
Oulasvirta et al. [16]	Movement	Throughput is calculated from mutual information of two or more deliberately repeated movement sequences.
Berdahl et al. [2]	Gesture	A model to account for human subject controlling a single, continuous sensor where the signal-to-noise ratio of the recorded gestures determines channel capacity.
Williamson and Murray-Smith [21] & Liu et al. [12]	Human Intention	Computer reduces the information-theoretic notion of uncertainty about the user intention.
Roy et al. [17] & Liu et al. [13]	Performance Measures	Using information-theoretic notions for measuring interaction techniques.
Zhang et al. [15]	Text Entry	A text entry throughput metric inspired by information-theoretic concepts.

CONCLUSION

The full spectrum of conceptual and practical benefits of Shannon's information theory seems to have a lot to offer to HCI. We hope that this paper will spur discussion and inspire more work that uses information theory to clarify our understanding of existing models and to design future user interfaces.

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REFERENCES

[1] Fred Attneave. 1959. Applications of information theory to psychology: A summary of basic concepts, methods, and results. (1959).

[2] Edgar Berdahl, Michael Blandino, David Baker, and Daniel Shanahan. 2016. An Approach for Using Information Theory to Investigate Continuous Control of Analog Sensors by Humans. In *Proceedings of the Audio Mostly 2016 (AM '16)*. ACM, New York, NY, USA, 85–90. https://doi.org/10.1145/2986416.2986450

[3] Stuart K Card, William K English, and Betty J Burr. 1978. Evaluation of mouse, rate-controlled isometric joystick, step keys, and text keys for text selection on a CRT. *Ergonomics* 21, 8 (1978), 601–613.

[4] Stuart K. Card, Allen Newell, and Thomas P. Moran. 1983. *The Psychology of Human-Computer Interaction*. L. Erlbaum Associates Inc., Hillsdale, NJ, USA.

[5] Andy Cockburn, Carl Gutwin, and Saul Greenberg. 2007. A Predictive Model of Menu Performance. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems (CHI '07)*. ACM, New York, NY, USA, 627–636. https://doi.org/10.1145/1240624.1240723

[6] Paul M Fitts. 1954. The information capacity of the human motor system in controlling the amplitude of movement. *Journal of experimental psychology* 47, 6 (1954), 381.

[7] Julien Gori, Olivier Rioul, and Yves Guiard. 2018. Speed-Accuracy Tradeoff: A Formal Information-Theoretic Transmission Scheme (FITTS). *ACM Trans. Comput.-Hum. Interact.* 25, 5, Article 27 (Sept. 2018), 33 pages. https://doi.org/10.1145/3231595

[8] William E Hick. 1952. On the rate of gain of information. *Quarterly Journal of Experimental Psychology* 4, 1 (1952), 11–26.

[9] Ray Hyman. 1953. Stimulus information as a determinant of reaction time. *Journal of experimental psychology* 45, 3 (1953), 188.

[10] William Lidwell, Kritina Holden, and Jill Butler. 2010. *Universal principles of design, revised and updated: 125 ways to enhance usability, influence perception, increase appeal, make better design decisions, and teach through design.* Rockport Pub.

[11] Wanyu Liu. 2018. *Information theory as a unified tool for understanding and designing human-computer interaction.* Ph.D. Dissertation. Université Paris-Saclay.

[12] Wanyu Liu, Rafael Lucas D'Oliveira, Michel Beaudouin-Lafon, and Olivier Rioul. 2017. BIGnav: Bayesian Information Gain for Guiding Multiscale Navigation. In *Proceedings of the 2017 CHI Conference on Human Factors in Computing Systems (CHI '17)*. ACM, New York, NY, USA, 5869–5880. https://doi.org/10.1145/3025453.3025524

[13] Wanyu Liu, Olivier Rioul, Michel Beaudouin-Lafon, and Yves Guiard. 2017. Information-Theoretic Analysis of Human Performance for Command Selection. In *IFIP Conference on Human-Computer Interaction*. Springer, 515–524.

[14] Scott MacKenzie and Poika Isokoski. 2008. Fitts' Throughput and the Speed-accuracy Tradeoff. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems (CHI '08)*. ACM, New York, NY, USA, 1633–1636. https://doi.org/10.1145/1357054.1357308

[15] Jacob O. Wobbrock. Mingrui "Ray" Zhang, Shumin Zhai. 2019. Text Entry Throughput: Towards Unifying Speed and Accuracy in a Single Performance Metric.. In *Proceedings of the 2019 CHI Conference on Human Factors in Computing Systems*. ACM, New York, NY, USA. https://doi.org/10.1145/3290605.3300866

[16] Antti Oulasvirta, Teemu Roos, Arttu Modig, and Laura Leppänen. 2013. Information Capacity of Full-body Movements. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems (CHI '13)*. ACM, New York, NY, USA, 1289–1298. https://doi.org/10.1145/2470654.2466169

[17] Quentin Roy, Yves Guiard, Gilles Bailly, Éric Lecolinet, and Olivier Rioul. 2015. Glass+ skin: an empirical evaluation of the added value of finger identification to basic single-touch interaction on touch screens. In *Human-Computer Interaction*. Springer, 55–71.

[18] Andrew Sears, Julie A Jacko, Josey Chu, and Francisco Moro. 2001. The role of visual search in the design of effective soft keyboards. *Behaviour & Information Technology* 20, 3 (2001), 159–166.

[19] Claude E Shannon. 1948. A mathematical theory of communication. 27 (1948), 379–423, 623–656.

[20] R William Soukoreff and I Scott Mackenzie. 1995. Theoretical upper and lower bounds on typing speed using a stylus and a soft keyboard. *Behaviour & Information Technology* 14, 6 (1995), 370–379.

[21] John Williamson and Roderick Murray-Smith. 2004. Pointing Without a Pointer. In *CHI '04 Extended Abstracts on Human Factors in Computing Systems (CHI EA '04)*. ACM, New York, NY, USA, 1407–1410. https://doi.org/10.1145/985921.986076

[22] Jacob O. Wobbrock and Brad A. Myers. 2006. From Letters to Words: Efficient Stroke-based Word Completion for Trackball Text Entry. In *Proceedings of the 8th International ACM SIGACCESS Conference on Computers and Accessibility (Assets '06)*. ACM, New York, NY, USA, 2–9. https://doi.org/10.1145/1168987.1168990

[23] Shumin Zhai. 2004. Characterizing computer input with Fitts' law parameters - the information and non-information aspects of pointing. *International Journal of Human-Computer Studies* 61, 6 (2004), 791–809.